1. a) Evaluate the cross-product $\hat{c} = \hat{a} \times \hat{b}$, where $\hat{a} = 3\hat{i} + 2\hat{j} + 1\hat{k}$ and $\hat{b} = 1\hat{i} + 2\hat{j} + 3\hat{k}$.
   
   $a/ \quad \hat{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 9(1-2) - 3(3-2) + 6(2-1) = 4\hat{i} - 9\hat{j} + 4\hat{k}$
   
   $b/ \quad |\hat{c}| = \sqrt{16 + 81} = \sqrt{97}$
   
   $|\hat{a}| = \sqrt{13}$, $|\hat{b}| = \sqrt{14}$, $\hat{a} \cdot \hat{b} = 6 + 4 + 3 = 13$
   
   $\cos \theta = \frac{121}{\sqrt{97} \sqrt{14}} = \frac{11}{\sqrt{13}}$

2. Two blocks of masses $M$ and $3M$ are placed on a horizontal, frictionless surface. A light spring is attached to one of them, and the blocks are pushed together with the spring between them. A cord initially holding the blocks together is burned; after this the block of mass $M$ moves to the left with left with speed $v$.

   a) Find the speed of mass $3M$.
   b) Find the original elastic potential energy of the spring in terms of the given quantities.

   \[ v_1 = \frac{-v}{2} \]

   \[ \frac{1}{2} kx^2 = \frac{1}{2} Mv^2 + \frac{3}{2} Mv_z^2 \]

   \[ x_0 = \frac{1}{2} \frac{Mv^2 + 4.7 \cdot 3}{Mv} \]

   \[ \left( x_0 \right)^2 = \frac{2}{3} Mv^2 \]
3. A billiard ball moving at 5 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves with a speed 4.33 m/s, at an angle of 30° with the original line of motion. Assuming an elastic collision (and ignoring friction and rotational motion) find the velocity of the struck ball after the collision.

\[ v_x = \frac{m \cdot u_x \cdot v}{m \cdot u_x + m \cdot u_y} \]
\[ v_y = \frac{m \cdot u_y \cdot v}{m \cdot u_x + m \cdot u_y} \]

\[ v_x = \frac{4.33 \cdot \cos 30°}{2} + \frac{5 \cdot \sin 30°}{2} = 3.79 \, \text{m/s} \]
\[ v_y = \frac{4.33 \cdot \sin 30°}{2} - \frac{5 \cdot \cos 30°}{2} = -1.25 \, \text{m/s} \]

\[ \sqrt{v_x^2 + v_y^2} = \sqrt{3.79^2 + (-1.25)^2} = 4.17 \, \text{m/s} \]

4. A uniform solid disk and a uniform hoop are placed side by side at the top of an incline of height \( h \). If they are released from rest and roll without slipping, which object reaches the bottom first? Verify your answer by calculating their linear speeds when they reach the bottom in terms of \( h \).

\[ mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \]
\[ \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(m+I/r^2)v^2 \]

\[ m = \frac{2mgh}{v} \quad I_{\text{hoop}} = \frac{1}{2}Mr^2 \rightarrow v^2 = \frac{2mgh}{I_{\text{hoop}}} = \frac{96}{\frac{3}{2}} = 48 \]
\[ I_{\text{disk}} = \frac{1}{2}Mr^2 \rightarrow v^2 = \frac{2mgh}{I_{\text{hoop}}} = \frac{48}{\frac{3}{2}} = 32 \]

The disk wins.
5. Consider the system illustrated above. The table surface is frictionless, the pulley has a mass $M$, a radius $R$ and is in the shape of a disk. At some instant in time, the masses are moving with a speed $v$ and an acceleration $a = \frac{dv}{dt}$.

a) Choosing the axis of the pulley as the axis of rotation, find the angular momentum of the system in terms of $v$ and the masses. (5 pts)

b) Find the time derivative of the angular momentum by differentiating your result in (a) (5 pts)

c) Find the torque on each mass and the pulley. Add them up to find the net torque $\sum \tau$ on the system. (5 pts)

d) Using Newton’s 2nd Law of (rotational) motion, find the acceleration $a$ of the masses. (5 pts)

e) Find the tensions $T_1$ and $T_2$ (10 pts)

\[ L = m_1 v R + m_2 v R + \frac{1}{2} I_{cm} \omega \]

\[ = \left( m_1 + m_2 + \frac{1}{2} M \right) v R \]

\[ \frac{dL}{dt} = \left( m_1 + m_2 + \frac{1}{2} M \right) \frac{dR}{dt} + \frac{1}{2} \frac{dI_{cm}}{dt} \omega \]

\[ \text{Torque on } m_1: \quad T_1 = -T_i R \]

\[ m_2: \quad T_2 = \frac{(T_i - T_1) R}{m_2 R} \]

\[ \theta = \frac{d\theta}{dt} = m_1 g R - T_1 R \]

\[ m_2: \quad \frac{\theta}{R} = \frac{m_2 g R}{m_2 R} \]

\[ \theta = \frac{m_2 g R}{m_2 R} \rightarrow a = \frac{m_2 g}{m_2 R} \]

\[ m_1 - m_2 - \frac{m_2 g R}{m_2 R} = m_1 a = \frac{m_1 (m_1 + m_2 + \frac{1}{2} M) \frac{dv}{dt}}{m_1 + m_2 + \frac{1}{2} M} \]

\[ a = \frac{m_1 g (m_1 + m_2 + \frac{1}{2} M)}{m_1 + m_2 + \frac{1}{2} M} \]

\[ T_2 = \frac{a}{m_2} = \frac{m_1 m_2 g}{m_1 + m_2 + \frac{1}{2} M} = \frac{2 m_1 m_2 g}{2 m_1 + 2 m_2 + m_1 + m_2 + \frac{1}{2} M} \]
6. A puck of mass $m$ is attached to a cord passing through a small hole in a frictionless, horizontal surface. The puck is initially orbiting with speed $v_1$ in a circle of radius $r_1$. The cord is then slowly pulled from below, decreasing the radius of the circle to $r_2 = \frac{r_1}{2}$.

a. What is the speed of the puck when $r = r_2$? (5 pts)

b. Calculate the ratio of the kinetic energies of the puck: $KE(r_2)/KE(r_1)$ (5 pts)

d. (Extra Credit) Find the tension in the cord as a function of $r$ (in terms of the initial conditions). (5 pts)

f. (Extra Credit) Using your answer to (d), find the work done in moving $m$ from $r_1$ to $r_2$. (5 pts)

\[ a. \quad m v_1 r_1 = m v_2 r_2 = m v_2 \frac{r_1}{2} \quad \Rightarrow \quad v_2 = 2v_1 \]

\[ b. \quad \frac{1}{2} m v_1^2 / \frac{1}{2} m v_2^2 = 4 \]

\[ d. \quad T = m \frac{v_1^2}{r_1} \quad \text{since} \quad v_{r_1} = v r \quad \Rightarrow \quad v^2 = \frac{v_{r_1}^2}{r_1^2} \]

\[ \Rightarrow \quad T = \frac{m v_{r_1}^2}{r_1} = T(r) \]

\[ f. \quad W = \int_{r_1}^{r_2} T \cdot dr = - \int_{r_1}^{r_2} T dr = - \int_{r_1}^{r_2} \frac{m v_{r_1}^2}{r_1^2} dr \]

\[ = \frac{m v_{r_1}^2}{r_1^2} \left[ \frac{r_2}{r_1} \right] = \frac{m v_{r_1}^2}{2} \left[ \frac{r_2}{r_1} - \frac{r_1}{r_2} \right] = \frac{3}{2} m v_{r_1}^2 \]