Possibly useful information:
Permittivity of free space: \( \varepsilon_0 = 8.85 \times 10^{-12} \text{F/m} \)  
Permeability of free space: \( \mu_0 = 4\pi \times 10^{-7} \text{T/m} \) 
Charge of a proton: \( e_p = 1.6 \times 10^{-19} \text{C} \) 
Proton mass: \( m_p = 1.67 \times 10^{-27} \text{kg} \) 
Charge of an electron: \( e_e = -1.6 \times 10^{-19} \text{C} \) 
Electron mass: \( m_e = 9.11 \times 10^{-31} \text{kg} \) 
\[ \int_x^y dx = \frac{x^n - y^n}{n+1} + C \text{ if } n \neq -1 \text{ and } = \ln x + C \text{ if } n = -1. \]

1. An electron with velocity \( \vec{v} = 6 \times 10^6 \text{ m/s} \) enters a region where the magnetic field is \( \vec{B} = 50 \times 10^{-4} \text{T} \).

   (a) Calculate the force \( \vec{F} \) the magnetic field exerts on the electron. (10 pts)

   (b) Calculate the radius of the electron’s orbit while it remains in this region. (10 pts)

   \[ \vec{F} = q_e \left( \vec{v} \times \vec{B} \right) = -1.6 \times 10^{-19} \begin{vmatrix} 0 & 0 & 0 \\ 5 \times 10^6 & 0 & 0 \\ 0 & 5 \times 10^6 & 0 \end{vmatrix} = -4.8 \times 10^{-13} \text{ N} \hat{z} \]

   \[ \vec{v} = \frac{m_e \vec{v}}{q_e} \rightarrow r = \frac{m_e \vec{v}}{q_e \vec{B}} = \frac{9.1 \times 10^{-31} \times 6 \times 10^6}{1.6 \times 10^{-19} \times 1.5 \times 10^{-5} \times 50 \times 10^{-4}} \approx 0.67 \text{ m} \]

2. A wire is formed into a circle having a diameter of 1 \( \times 10^{-2} \text{ m} \) and is placed in a magnetic field of \( B = 3 \times 10^{-7} \text{T} \). The wire carries a current of 1 \( \text{A} \).

   (a) Find the magnetic moment of this loop. (5 pts)

   (b) What is the maximum torque this loop can experience in the given field? (5pts)

   \[ \vec{M} = IA = I \left( \frac{1}{2} \vec{A} \times \vec{B} \right), \left| A \right| = 5.85 \times 10^{-5} \text{ A} \cdot \text{m}^2 \]

   \[ \vec{\tau}_{\text{max}} = \vec{M} \times B = 7.85 \times 10^{-5} \text{ A} \cdot \text{m}^2 \times 3 \times 10^{-7} \text{T} = 2.3 \times 10^{-9} \text{ N} \cdot \text{m} \]
3. The segment of wire in the figure below carries a current of $1 \, A$. The radius of the circular arc is $R = 8 \, cm$. Determine the magnitude and direction of the magnetic field at the origin. (15 pts)

\[ \text{But Saint-Venant:} \quad d \mathbf{B} = \frac{M_0}{4\pi} \frac{\mathbf{E} \times \mathbf{r} \times \mathbf{r}}{r^3} \]

By inspection, the only contribution is along the circular arc, where $\mathbf{F} \cdot \mathbf{r} = \mathbf{I} \mathbf{R} \mathbf{d} \theta$ (into the page).

Then \[ B = \frac{M_0 I}{4\pi} \int_{0}^{\alpha} \frac{r_0 k \mathbf{r}}{R} = \frac{M_0 I}{4\pi} \frac{I}{R} \frac{r_0}{R^2} \]

\[ B = \frac{M_0 I}{4\pi} \frac{r_0}{R} \]

Into the paper:

\[ B = \frac{0.19 \times 10^{-7} \, T}{1 \times (0.08)} = 1.98 \times 10^{-5} \, T \]

4. In the figure below, (a) calculate the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire. (b) Calculate the magnetic flux through the loop due to $I_2$. (15 + 10 pts)

\[ I_2 \]

\[ \text{a) Force for unit length: } \mathbf{F} = \frac{2M_0 I_2}{2} \left( \frac{1}{a} - \frac{1}{b} \right) \quad \text{(observe if current is parallel)} \]

\[ \mathbf{F} = \frac{2M_0 I_2}{2\pi} \left( \frac{1}{a} - \frac{1}{b} \right) \]

\[ \text{Since } \mathbf{x} = C, \]

\[ \mathbf{F} = \frac{-b - c + \frac{1}{2\pi} (a + b)}{2\pi (a + b)} \text{ (torque on wire)} \]

\[ \text{b) Flux: } \Phi = \frac{2M_0 I_2}{2\pi} \left( \frac{a}{a} \right) \]

\[ \Phi = \frac{2M_0 I_2}{2\pi} \left( \frac{a}{a} \right) \]
5. A coil of 20 turns and radius 10 cm surrounds a long solenoid of radius 2 cm and \(1 \times 10^3\) turns per meter, if the current in the solenoid changes as \(I = 5 \sin(120t)\), find the induced emf as a function of time. (15 pts)

\[
\text{emf} = -N \frac{d\Phi_B}{dt} = -N \frac{d}{dt} \left( B \pi r^2 \right) = -NA \frac{dI}{dt}
\]

\[
= -N \pi r^2 \frac{d}{dt} \left( I \mu_0 \pi l \right)
\]

\[
= -N \pi r^2 \mu_0 \pi l \frac{dI}{dt} \]

\[
= 20 \pi (0.02)^2 \mu_0 \pi l \frac{dI}{dt} \]

\[
= \frac{20 \pi (0.02)^2 \mu_0 \pi l \frac{dI}{dt}}{600 \cos(120t) \times 4 \times 10^{-7} \times 1 \times 10^3}
\]

\[
= -0.019 \cos(120t) \times 4 \times 10^{-7} \times 1 \times 10^3
\]

6. A 0.15 kg wire in the shape of a closed rectangle is 1 m wide and 1.5 m long and has a total resistance of 0.75 \(\Omega\). The rectangle is allowed to fall through a magnetic field directed perpendicular to the direction of motion of the rectangle. The rectangle accelerates downward as it approaches a terminal speed of \(v_t = 2 \text{ m/s}\), with its top not yet in the region. Calculate the magnitude of \(B\). (15 pts).

\[
mg = \beta I v
\]

\[
\Rightarrow \beta = \frac{mg}{v} = \frac{\frac{1}{2}m v_t}{v} = \frac{0.15}{2} = 0.75
\]

\[
\Rightarrow B = \frac{B v_0}{\sqrt{v_t^2 - v^2}} = \frac{0.75 \times 2}{\sqrt{4^2 - 2^2}} = \frac{1.5}{2}
\]

\[
B = 0.742 \text{ T}
\]