1. A solid cube of silver (mass density $\rho = 10.6 \text{ g/cm}^3$) has a volume of $8.57 \text{ cm}^3$. The resistivity of silver is $\rho = 1.59 \times 10^{-4} \Omega \cdot \text{m}$, and there are $5.02 \times 10^{22}$ silver atoms present. If there is one conduction electron per atom, and if a potential difference of $1 \times 10^{-3}$ volts is applied between opposite faces of the cube, find

a. the resistance $R$ of the cube, (6 pts)
$$R = \rho \frac{L}{A} = (1.59 \times 10^{-4} \Omega \cdot \text{m}) \frac{2.05 \text{ cm}}{8.57 \times 10^{-4} \text{ cm}^2} = 9.36 \times 10^{-2} \Omega$$

b. the electric field $E$ inside the cube, (5 pts)
$$E = \frac{V}{\frac{L^2}{2}} = \frac{4.18 \times 10^{-4} \text{ V/m}}{2.05 \text{ cm}^2} = 2.01 \times 10^{-4} \text{ V/cm}$$

c. the number density $n$ of the charge carriers, (6 pts)
$$n = \frac{e}{e} = \frac{1.6 \times 10^{-19} \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 1 \times 10^{23} \text{ m}^{-3}$$

d. the conductivity $\sigma$ of the cube, (5 pts)
$$\sigma = \frac{1}{\rho} = \frac{1}{6.27 \times 10^{-4} \Omega \cdot \text{m}} = 1.6 \times 10^{-22} \text{ S/m}$$

e. the current density $J$ in the cube, and (5 pts)
$$J = \sigma E = 1.6 \times 10^{-22} \text{ S/m} \times 2.01 \times 10^{-4} \text{ V/cm} = 3.22 \times 10^{-26} \text{ A/m}^2$$

f. the average drift speed $v_d$ of the electrons in the cube. (5 pts)
$$v_d = \frac{J}{ne} = \frac{1.6 \times 10^{-22} \text{ S/m}}{1.6 \times 10^{-19} \text{ C}} = 3.22 \times 10^{-6} \text{ m/s}$$
2. Derive the capacitance for a parallel plate capacitor filled with two dielectrics in parallel. The distance between the two plates is \( d \), the plate surface area is \( A \), the surface area of the dielectric with dielectric constant \( \kappa_1 \) is \( \frac{A}{3} \), and the surface area of the dielectric with dielectric constant \( \kappa_2 \) is \( 2A/3 \). (20 pts)

\[
C = C_1 + C_2 = \frac{\kappa_1 \varepsilon_0 A}{d} + \frac{\kappa_2 \varepsilon_0 A}{3d} = \frac{\varepsilon_0 A}{d} \left( \frac{\kappa_1}{3} + \frac{\kappa_2}{2} \right)
\]

3. A parallel-plate capacitor of surface area \( A \) and separation distance \( d \) is charged to a potential difference \( \Delta V \), with a charge \( +Q \) on one plate and \( -Q \) on the other. The capacitor remains connected to the battery and a conducting slab of thickness \( d/2 \) and cross-section area \( A \) is inserted in the space in the middle of the capacitor. Find:

(a) the new potential difference between the capacitor plates, (10 pts) and

(b) the new capacitance of the plates + slab, (10 pts)

\[
V' = \frac{Q}{C'}
\]

(c) The battery maintains a potential difference of \( D V' \)

\[
C' = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{\varepsilon_0 A}{d} \left( \frac{\kappa_1}{3} + \frac{\kappa_2}{2} \right)
\]

\[
C_1 = \frac{\varepsilon_0 A}{d} \left( \frac{1}{\kappa_1} + \frac{1}{\kappa_2} \right) = \frac{\varepsilon_0 A}{d} \left( \frac{\kappa_1 + \kappa_2}{\kappa_2} \right)
\]

\[
C_2 = \frac{\varepsilon_0 A}{d} \left( \frac{1}{\kappa_1} + \frac{1}{\kappa_2} \right) = \frac{\varepsilon_0 A}{d} \left( \frac{\kappa_1 + \kappa_2}{\kappa_1} \right)
\]

\[
C' = \frac{(4\varepsilon_0 A/3)^2}{4\varepsilon_0 A/3 + 4\varepsilon_0 A/3} = \frac{16\varepsilon_0 A}{6A} = \frac{8\varepsilon_0}{3}
\]
4. Find the current in each branch and the power dissipated in each resistor in the circuit above. (15 pts)

\[ A_1 V = \frac{V \cdot V}{W} = 2R \]

\[ \text{Loop 1: } 50 - 2I_1 - 2I_2 = 0 \quad 2I_1 + 2I_2 = 50 \]

\[ \text{Loop 2: } 70 - 2I_2 + 2I_3 = 0 \quad 2I_2 - 2I_3 = 70 \]

\[ \text{Node: } I_1 = I_2 + I_3' \quad I_3' = I_3 - I_2 \]

\[ 2I_1 + 2(I_2 - I_3) = 50 \]

\[ 4I_1 - 2I_2 = 50 \]

\[ 2I_2 - 2(I_2 - I_3) = 70 \]

\[ 4I_2 - 2I_3 = 70 \]

\[ 8I_3 - 4I_1 = 80 \]

\[ 6I_3 - 90 = 15A \]

\[ 4I_3 - 20 = 2I_1 \]

\[ 2I_3 - 10 = \vec{I}_3 = 2.15 - 10 = 70 \]

\[ \vec{I}_2 = 70 - 15 = 55 \]

\[ \vec{I}_3 = \vec{I}_2 + \vec{I}_4 \quad \vec{I}_3 = \vec{I}_2 + \vec{I}_4 \]

\[ (\text{Note: } I_3 = 8A) \]
In the circuit below, find the equivalent resistance and total current between points a and b. The potential difference between a and b is 10 volts. (15 pts)

\[
\begin{align*}
\text{Loop 1:} & \quad 10 - 2I_1 + (5)I_1 = 0 \\
& \quad 10 - 3I_1 - I_2 = 0 \\
L1: \quad I_2 = 0 - 3I_1 \\
& \quad I_1 + I_2 = 10 \\
& \quad I_1 - 3I_1 - 6I_2 = 10 \\
& \quad I_1 - 3I_1 - 60 + 18I_1 = 10 \\
& \quad I_1 + 15I_1 = 70 \\
& \quad \text{Eliminate } I_1: \quad 75I_1 + 10 I_2 = 490 \\
& \quad 120I_1 + 165I_1 = 980 \\
\Rightarrow & \quad I_2 = \frac{490}{120} = 4.0833 \\
\Rightarrow & \quad I_1 = \frac{980 - 490}{120} = 4.0833 \\
\Rightarrow & \quad \text{Equivalent resistance: } R_{eq} = \frac{V}{I} = \frac{10}{4.0833} = \frac{850}{34} \Omega \\
& \quad \frac{850}{34} \Omega = 2.529 \Omega
\end{align*}
\]