Physics 216 Problem Set #5 - Solutions

1. (a) The Coulomb force exerts a centripetal force on the electron: \( \frac{kp^2}{r^2} = m \frac{v^2}{r} \)
   
   so \( v = \sqrt{\frac{kp^2}{mr}} = \sqrt{\frac{9 \times 10^9 (1.6 \times 10^{-19})^2}{9.11 \times 10^{-31} \times 5.29 \times 10^{-11}}} = 2.19 \times 10^6 \text{m/s} \)

   (b) The frequency of revolution is \( f = \frac{v}{2\pi r} = \frac{2.19 \times 10^6 \text{m/s}}{2\pi \times 5.29 \times 10^{-11} \text{m}} \approx 6.59 \times 10^{15} \text{s}^{-1} \).
   
   The current is then \( i = qf = (1.6 \times 10^{-19} \text{C}) \cdot (6.59 \times 10^{15} \text{s}^{-1}) = 1.05 \times 10^{-3} \text{A} \)

2. \( Q = \int I(t) \, dt = \int_0^{240} 100 \sin(120\pi t) \, dt = \frac{100}{120\pi} \int_0^{\frac{\pi}{3}} \sin u \, du = \frac{100}{120\pi} (-\cos \frac{u}{3}) = \frac{5}{6\pi} \text{C} \)

3. Here \( \frac{\Delta q}{\Delta t} = \frac{q}{T} = qf = \frac{q\omega}{2\pi} \)

4. We have \( R = \rho \frac{l}{A} = 0.500 \Omega \) and a volume \( V = \frac{1.000 \times 10^{-3} \text{kg}}{8.92 \times 10^3 \text{kg/m}^3} = 1.12 \times 10^{-7} \text{m}^3 \)
   to work with. Now \( V = Al \) and \( \frac{l}{A} = \frac{8}{\rho} = \frac{0.500 \Omega}{1.7 \times 10^{-1} \Omega \cdot \text{m}} = 2.94 \times 10^7 \text{m}^{-1} \) so
   
   \( l = A \cdot 2.94 \times 10^7 \text{m}^{-1} \). Substitution into \( V = Al \) gives \( 1.12 \times 10^{-7} \text{m}^3 = A^2 \cdot 2.94 \times 10^7 \text{m}^{-1} \). Solving for \( A \) gives \( A = 6.17 \times 10^{-8} \text{m}^2 \)
   Equating this result with \( \pi \left( \frac{d}{2} \right)^2 \) and solving for \( d \) gives \( d = 2.80 \times 10^{-4} \text{m} \). Also, \( l = A \cdot 2.94 \times 10^7 \text{m}^{-1} = 6.17 \times 10^{-8} \text{m}^2 \cdot 2.94 \times 10^7 \text{m}^{-1} = 1.81 \text{m} \)

5. (a) The charge carrier density remains the same since the substance remains the same. (b) Doubling the current doubles the current density, assuming the cross-sectional area doesn’t change. (c) The drift velocity is doubled, as can be inferred from \( \mathbf{J} = nq\mathbf{v} \).
   (d) The mean-free path \( \tau = l/\mathbf{v} \) stays the same since \( \mathbf{v} \) depends on temperature but not on the applied field.

6. Neglecting the changes in length and area of the wire, we use \( R = R_0 [1 + \alpha (T - T_0)] \). After some algebra we get \( T = T_0 + \frac{R - R_0}{4.5 \times 10^{-3}} \approx 2.7 \times 10^4 \text{C} \)

7. The resistance is \( R = \rho \frac{l}{A} = (1.5 \times 10^{-6} \Omega \cdot \text{m}) \frac{25 \text{m}}{\pi (0.4 \times 10^{-3} / 2 \text{m})^2} = 298 \Omega \).
   The potential difference across the wire is 298 \( \Omega \cdot 0.5 \text{A} \) = 149 \( V \).
   Then \( E = V/I = 149 V/25 m = \frac{149 V}{25 m} \).
   (b) The power is \( P = (0.5)^2 (298) = 47 \text{ Watts} \)
   (c) The resistance becomes \( R = R_0 [1 + \alpha (T - T_0)] = 298 [1 + 0.4 \times 10^{-3} (340 - 20)] = 336 \Omega \), so that the current drawn through it is \( \frac{149}{336} = \frac{0.443}{A} \). Then the power is \( IV = 66 \text{ Watts} \).
8. The cross-section area of the transmission line is 

\[ A = \pi \left( \frac{2 \times 10^{-2} m}{2} \right)^2 = 3.14 \times 10^{-4} m^2. \]

The current density \( J = \frac{I}{A} = \frac{1000}{3.14 \times 10^{-4} m^2} = 3.18 \times 10^6 A/m^2 \)

\[ = nqv_d. \]

Then \( v_d = \frac{J}{nq} = \frac{3.18 \times 10^6}{8 \times 10^{28} \cdot 1.6 \times 10^{-19}} = 2.5 \times 10^{-4} m/s. \) Therefore the time required to traverse the line is 

\[ t = \frac{L}{v_d} = \frac{200 \times 10^3}{2.5 \times 10^{-4}} = 8 \times 10^8 s \approx 25.5 \text{ yr}. \]