\[ b(x) = \begin{cases} 
-1, & x < 2 \\
1, & x = 2 \\
3, & x > 2 
\end{cases} \]

What is \( \lim_{x \to 2} b(x) \)?

Since the "left limit" and "right limit" are different,

\( \lim_{x \to 2^-} b(x) \) Does Not Exist (DNE)

Left hand limit: \( \lim_{x \to 2^-} b(x) = -1 \)

Right hand limit: \( \lim_{x \to 2^+} b(x) = 3 \)
\[
\lim_{x \to -4} \frac{x^2 + 10x + 24}{x + 4} = \frac{(-4)^2 + 10(-4) + 24}{-4 + 4} = \frac{0}{0}
\]

indeterminate form

\[
\lim_{x \to -4} \frac{(x+4)(x+6)}{x+4} = \lim_{x \to -4} x+6 = -4+6 = 2
\]

Continuity Revisited:

1. No holes

\[
\begin{array}{ccc}
\text{No holes} & \text{No jumps} & \text{No gaps} \\
\text{C} & \text{C} & \text{C}
\end{array}
\]

For continuity of \( f \) at \( x = c \):

1. \( f(c) \) exists
2. \( \lim_{x \to c} f(x) = f(c) \)
3. \( \lim_{x \to c} f(x) = f(c) \)
2.1 Rate of change

"slope"

Example: I drive 360 miles in 6 hours.

Rate of change = speed = \frac{360 \text{ miles}}{6 \text{ hours}}

= 60 \text{ mph}

Average Speed / Velocity

Speed vs. Velocity

Speed: constant, scalar
always positive

Velocity: magnitude & direction (vector!)
Positive / negative
Drop an object from 10 ft.

<table>
<thead>
<tr>
<th>time (secs)</th>
<th>0</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.79</th>
</tr>
</thead>
<tbody>
<tr>
<td>height (ft)</td>
<td>10</td>
<td>6</td>
<td>4.24</td>
<td>2.16</td>
<td>0</td>
</tr>
</tbody>
</table>

Find the average velocity:

a) $\frac{10-0}{0-0.79} \approx -12.7$

b) $-8$

c) $-19.2$

d) $-17.6$

Imagine $v(t)$ gives the instantaneous velocity after $t$ seconds. $v(0) = 0 \text{ ft/sec}$

$v(0.5) \approx -17.6 \text{ ft/sec}$

Could we find a better approximation?

Yes! use an even closer interval of time:

- Example: 0.5 to 0.501 sec
- Example: 0.499 to 0.501 sec
Instantaneous Rate of Change:

**Graphically:**

\[ f(a+h) \]
\[ f(a) \]
\[ \text{slope of secant line} = \frac{\text{rise}}{\text{run}} \]
\[ = \frac{f(a+h) - f(a)}{h} \]

To approximate the instantaneous rate of change at \( x=a \), use the slope of the secant line.

To find the exact rate of change, take a limit as \( h \to 0 \):

\[ \text{slope of the tangent line at } x=a \]
Show that the slope of the tangent line of \( y = x^2 \) at \( x = 3 \) is 6 by evaluating:

\[
\lim_{h \to 0} \frac{(3+h)^2 - (3)^2}{h} = \lim_{h \to 0} \frac{9+6h+h^2 - 9}{h}
\]

\[
= \lim_{h \to 0} \frac{6h+h^2}{h} = \lim_{h \to 0} h(6+h)
\]

\[
= \lim_{h \to 0} (6+h) = 6 + 0 = 6
\]
2.2 Derivative at a point

The derivative of $f(x)$ at $x = a$ is notated by $f'(a)$ ("f prime of a") and it gives the slope of the tangent line at $x = a$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$
Estimate \( f'(2) \) for \( f(x) = x^x \). Use \( h = 0.001 \)

\[
f'(2) \approx \frac{f(2 + 0.001) - f(2)}{0.001}
\]

\[
= \frac{f(2.001) - f(2)}{0.001}
\]

\[
= \frac{2.001^2 - 2^2}{0.001}
\]

\[
= \frac{4.006779 - 4}{0.001} = 6.779
\]