1.2 cont.

Concavity: " is a function's graph curving upward or downward?"

Linear function
no concavity

4 instances:

1
2
concave up
concave down
concave up
concave down

3
4
+ increasing
+ increasing
+ decreasing
+ decreasing
New Functions from Old Functions

\[ y = x^2 \]

**BASIC GRAPH**

Now graph:
\[ y = x^2 - 5, \quad y = x^2 + 1 \]

**shift down 5** **shift up 1**

What about:
\[ y = (x-5)^2, \quad y = (x+1)^2 \]

**shifts right 5** **shifts left 1**

**BASIC GRAPH:**
\[ y = \sqrt{x} \]

How are the graphs of \( y = \sqrt{-x} \) and \( y = -\sqrt{x} \) different?

If \((9,3)\) is on the graph of \( y = \sqrt{x} \),
\((-9,3)\) is on the graph of \( y = \sqrt{-x} \).

\[ y = \sqrt{-x} \] is a horizontal reflection of \( y = \sqrt{x} \).

\[ y = -\sqrt{x} \] is a vertical reflection of \( y = \sqrt{x} \).

What is the domain and range of \( y = -\sqrt{x} \)?

**D:** \((-\infty, 0] \]

**R:** \((-\infty, 0] \]
Transformations:

If it is next to the $x$, then it happens in the $x$-direction (horizontally), and it does the opposite of what you might expect.

If it is outside the function (not next to the $x$), then it happens in the $y$-direction (vertically), and it does what you would expect.

**Basic Graph**: $y = x^3$

How are the following graphs different?

a) $y = \frac{1}{8}x^3$
   - Vertical compression by a factor of $\frac{1}{8}$

b) $y = 125x^3$
   - Vertical expansion/stretch

c) $y = \left(\frac{1}{2}x\right)^3$
   - Horizontal expansion/stretch

d) $y = (5x)^3$
   - Horizontal compression

**Note**: $\left(\frac{1}{2}x\right)^3 = \frac{1}{8}x^3$ and $(5x)^3 = 125x^3$
(Ex) \( y = 2x^2 + 12x - 1 \)

a parabola, so a transformation of \( y = x \)

**Complete the square first:**

\[
y = 2\left(x^2 + 6x + 9\right) - 1 - 2(9)
\]

\[
\frac{6}{2} = 3
\]

\[
3^2 = 9
\]

\[
y = 2(x+3)^2 - 19
\]

**3 transformations:**

2: vertical expansion
3: shift left 3
-19: shift down 19
\[ y = 20 + 17e^{-0.063t} \]

is a transformation of the basic graph \[ y = e^t \]

- **Horizontal reflection**
- **Vertical expansion**

\[ y = 17e^{-0.063t} \]

\[ y = 20 + 17e^{-0.063t} \]

- **Horizontal asymptote**
- **Vertical shift up 20**
Symmetry:

Palindrome: MOM, RACECAR

A NUT FOR A JAR OF TUNA

"Function symmetry:"

1. **EVEN function**: \( f(-x) = f(x) \)
   - Symmetry about the y-axis

2. **ODD function**: \( f(-x) = -f(x) \)
   - Symmetry about the origin
     if \((a, b)\) is a point, so is \((-a, -b)\)

3. **Example**: \( g(x) = \frac{|x|}{x} - 144x^3 \)

   **Test**: \( g(-x) = \frac{|-x|}{-x} - 144(-x)^3 \)
   
   \[ = \frac{|x|}{-x} - 144(-x^3) \]
   
   \[ = \frac{|x|}{-x} + 144x^3 \]
   
   \[ = -\left(\frac{|x|}{x} - 144x^3\right) \]
   
   \[ = -g(x) \]
Composition of functions:

Given two functions \( f \) and \( g \),

1. \((f \circ g)(x) = f(g(x))\)
2. \((g \circ f)(x) = g(f(x))\)

A spherical balloon is being filled with helium in such a way that its radius is increasing by 2 cm every second. Find:

a) a function which gives the radius of the balloon after \( t \) seconds
b) a function which gives the volume when the radius is \( r \).

c) Find \((V(\circ r))(t)\) and interpret.

c) \(V(r(t)) = V(2t) = \frac{4}{3}\pi(2t)^3\)

\[ V(t) = \frac{32}{3}\pi t^3 \] which gives the volume of the balloon after \( t \) seconds have passed.
Given the graphs of \( y = g(x) \) and \( y = f(x) \) in the following figure, estimate \( f(g(-5)) \).

\[(f \circ g)(-5) = f(g(-5)) = f(0) = -15\]

Find \( f \) and \( g \) given that:

\[h(x) = (f \circ g)(x) = \frac{2}{7-x^3}\]

\[f(x) = \frac{2}{x}, \quad g(x) = 7-x^3, \quad f(x) = \frac{2}{7-x^3}, \quad g(x) = x^3\]

\[f(x) = 2x, \quad g(x) = \frac{1}{7-x^3}\]

\[f(x) = \frac{2}{7+x}, \quad g(x) = -x^3\]
**Inverses:**

If \( y = b(x) \), then \( x = b^{-1}(y) \)

For instance: \( F(C) \) gives °F

For an input of °C

\( C(F) \) is its inverse

(input & output switch) (Domain & Range switch)

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To find \( b^{-1} \) given \( b \):

1. Switch all \( x \)'s and \( y \)'s
2. Solve for \( y \)

3. Find \( b^{-1} \) if \( f(x) = \frac{1}{x+y} \)

\[
\begin{align*}
  y &= \frac{1}{x+y} \\
  \Rightarrow x &= \frac{1}{y} \\
  \Rightarrow x(x+y) &= 1
\end{align*}
\]

\[
\begin{align*}
  \Rightarrow y + y &= \frac{1}{x} \\
  \Rightarrow y &= \frac{1}{x} - y \\
  \Rightarrow b^{-1}(x) &= \frac{1}{x} - y
\end{align*}
\]

**NOTE:**

\( f \): 1st - odd \( y \)  \( f^{-1} \): 1st - invert

2nd - invert  2nd - subtract \( y \)
If the point \((12, -5)\) is on the graph of \(y\), what point must lie on:

a) \(y^{-1}\)  

b) \(y\), if \(y\) is even

c) \(y\), if \(y\) is odd

d) \(2y - 10\)

a) \((-5, 12)\)  (input \& output switch!

b) \((-12, -5)\)  (horizontal symmetry!)

c) \((-12, 5)\)  (symmetry about \((0,0)\))

d) \((12, -20)\)  (output is doubled then 10 less)