EXPERIMENT 9

Collisions and Linear Momentum

OBJECTIVE:
To introduce the concept of linear momentum, the product of mass and velocity; to investigate the relation between force and momentum and the usefulness of momentum in describing collisions.

INTRODUCTION:
Momentum is another of those concepts which seems somewhat nebulous and abstract to the beginning physics student. Almost every student has, at sometime or another, exclaimed “Yes, I KNOW it’s mass times velocity, but WHAT is that, really?” only to be frustrated by the instructor’s perennial reply, “But that’s what it is, REALLY.”

Perhaps it’s easier to accept momentum as a concept if you think of it in the same way in which you accepted the concept of work. Physicists define it the way they do because it works. This rather odd quantity can be used in another powerful conservation equation, the CONSERVATION OF LINEAR MOMENTUM.

Every object with mass and velocity has associated with it a “quantity of motion” called MOMENTUM. The momentum of a moving particle is directly proportional both to its mass and to its velocity. Momentum is a VECTOR quantity having the same direction as velocity.

Momentum is extremely important in collision problems. When two objects collide, each exerts a force on the other. The product of the FORCE and the TIME for which it acts is another useful quantity called the IMPULSE of the force. An alternate way of writing Newton’s Second Law of Motion is to state that the IMPULSE of the force on an object is equal to the CHANGE in MOMENTUM of that object.

\[ \text{Impulse} = F \times t = \Delta p = m\Delta v. \]  \hspace{1cm} (9.1)

When two or more objects interact with each other, they are said to comprise a SYSTEM. An ISOLATED SYSTEM is a system which does not exchange energy with its environment. In this case there is no net external force acting on the system or its components. (However, the components may exert internal forces on each other - by Newton’s Third Law these internal forces add up to zero for the system as a whole.) Therefore there is NO CHANGE IN MOMENTUM for an isolated system, and we say that the MOMENTUM OF AN ISOLATED SYSTEM IS CONSERVED. This is the LAW OF CONSERVATION OF LINEAR MOMENTUM, and it is a useful tool in solving problems of collision and recoil. While the total momentum of an isolated system remains constant, the momenta of the individual components of the system CAN and DO change.
While the law of conservation of momentum holds in EVERY collision for an isolated system, certain collisions occur where kinetic energy is also conserved. These collisions are called ELASTIC: there is no permanent deformation of the objects and all the kinetic energy possessed by the system before collision is retained by the system after the collision. Collisions which do not conserve kinetic energy are called INELASTIC collisions. Elastic collisions are the subject of this experiment – inelastic collisions are the subject of the next.

Suppose we have a system of two objects of masses \(m_1\) and \(m_2\), respectively, traveling horizontally toward each other en route to a collision. In one dimension their initial respective velocities are \(v_1\) and \(v_2\) before the collision. After the collision their velocities are \(u_1\) and \(u_2\).

Conservation of momentum says that the initial momentum of the system equals the final momentum of the system:

\[
m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2.
\]  
(9.2)

In elastic collisions, no kinetic energy is lost; therefore

\[
\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2.
\]  
(9.3)

The algebra is tough, but using these last two equations you can show that

\[
u_1 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2
\]  
(9.4)

and

\[
u_2 = \left(\frac{2m_1}{m_1 + m_2}\right)v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2
\]  
(9.5)

Note that the \(u\)'s and the \(v\)'s can be negative. That's because they are velocities (vectors) instead of speeds (numbers).

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**DEMONSTRATION STUDIES:**

1. One-Dimensional Collisions
2. Elastic Collisions
3. The Astroblaster ®

**EQUIPMENT NEEDED**

1. Air track w/ gliders
2. Momentum transfer toy
3. Astroblaster ®, goggles, air track

**PLEASE ANSWER ALL OBSERVATION QUESTIONS ON YOUR DATA SHEETS USING COMPLETE SENTENCES.**

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\[.2\]
STUDY #1 – ONE DIMENSIONAL COLLISIONS BETWEEN TWO OBJECTS

1. You will use an air track without the spark timer for this study. Two gliders will collide on the frictionless track providing an isolated system.

2. Weigh the small glider and one of the large gliders. Record their masses (OQ 1a).

3. Place the small glider at rest on the center of the air track. Place the large glider about a half a meter to the left of the small glider.

4. You will gently shove the large glider toward the small glider. Before you do this, guess the direction that the large glider will go after it collides. To help guide your thinking, set $v_2 = 0$ in (9.4). Will it travel in the same direction as before, the opposite direction, or stop altogether? Record your guess in (OQ 1b).

5. Turn on the air source and launch the other glider as to collide it with the stationary glider. Observe the subsequent motion of each glider (no numerical values are needed here). Did your results confirm your expectation? (OQ 1c).

6. Switch the positions of the gliders and repeat steps 4 & 5. This time the small glider approaches the stationary large glider (OQ 1d & 1e).

7. Finally, repeat steps 4 & 5 using the two large gliders. Again, guess the result (OQ 1f) and confirm it (OQ 1g).

8. Newton originally expressed his Second Law of Motion like this: $F_{net} = \frac{\Delta p}{\Delta t}$.

   Was there a force acting on the stationary glider during the collision? (OQ 1h)

9. Recall Newton's Third Law of Motion. Was there a force acting on the moving glider? (OQ 1i)

\[ U_i \quad \rightarrow \quad U_f \]

\[ \begin{array}{c|c}
& \\
\text{Before} & \\
\hline
\end{array} \]

\[ \begin{array}{c|c}
& \\
\text{After} & \\
\hline
\end{array} \]
STUDY #2 – ELASTIC COLLISIONS

1. You will use the momentum transfer toy for this study. The collisions between the balls of this toy are elastic, so kinetic energy is conserved.

2. Pull one of the masses out and allow it to collide with the remaining four balls. Observe what happens. Pull out two masses and repeat. Then repeat with three masses. Note that all the masses are equal.

![Diagram of balls and strings]

OBSERVATION QUESTION 2

a) Describe what happens in each case.
b) Explain what you observed in terms of momentum.
c) When you allowed two masses to collide with the rest of the balls, two masses took off after the collision with approximately the same velocity as the incoming balls. Would it be possible for ONE ball to fly off with TWICE the velocity of the incoming balls? (Hint: to investigate momentum conservation, set $(2m) v = m(2v)$ and check for equality. Next, compare the kinetic energy of the system before and after the collision.)

![Diagram of two masses and strings]

![Diagram of single mass and strings]

d) If you let the balls continue to collide with each other, the system eventually comes to rest. This indicates that some energy is lost during the collisions (i.e., the collection isn’t “perfectly” elastic). Where does the energy go?
STUDY # 3 – THE ASTROBLASTER®

Recall that the equations for the velocities of the objects after a two-body collision in one dimension are:

\[ u_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_1 + \left( \frac{2m_2}{m_1 + m_2} \right) v_2 \quad \text{and} \quad u_2 = \left( \frac{2m_1}{m_1 + m_2} \right) v_1 + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) v_2 \]

where \( v_1 \) and \( v_2 \) are the initial velocities of the object. We consider two special cases.

I. An object bouncing off a ground or wall.
Suppose \( v_2 = 0 \) and \( m_2 \gg m_1 \). Then the above equations reduce to \( u_1 = -v_1 \) and \( u_2 = 0 \). (Check this yourself.) This shows that the recoil speed of an object bouncing off the floor or wall is that same as its initial incoming speed. Here \( m_2 \) is the mass of the Earth plus wall.

II. A small object bouncing off a larger recoiling object
Now suppose an object with mass \( m_3 \), with \( m_3 < m_1 \), is trailing behind the first object. The first object collides with \( m_2 \) (the Earth or the wall) and hits the third object on its rebound. Let \( v_3 \) be the speed of the third object before collision and \( w_3 \) be the speed of the third object after hitting the first object on its rebound. Again supposing a stationary earth or wall we have \( u_1 = -v_1 \) and \( u_2 = 0 \). Then

\[ w_3 = \left( \frac{m_3 - m_1}{m_1 + m_3} \right) v_5 + \left( \frac{2m_1}{m_1 + m_3} \right) u_1 = \left( \frac{m_3 - m_1}{m_1 + m_3} \right) v_5 - \left( \frac{2m_1}{m_1 + m_3} \right) v_1 \]

If the incoming velocities are the same (i.e. \( v_1 = v_3 \)), then the velocity of the third object after the collision is

\[ w_3 = \left( \frac{m_3 - m_1}{m_1 + m_3} \right) v_1 \]

Because \( m_3 < m_1 \), the recoil speed of the third mass will be quite large! (Using a basketball for \( m_1 \) and a tennis ball for \( m_3 \) demonstrates this very nicely)

**OBSERVATION QUESTION 3**

(a) On the air track, send the small glider and the large glider together toward the end of the track closest to the large glider. Compare the speeds of the small glider before and after the collision with the track end.

(b) (Optional) Have the Lab supervisor demonstrate the Astroblaster. This little device suggests how cosmic rays are accelerated to very high speeds via collisions with other cosmic particles. (No questions or answers – just for fun.)
SUMMARY:

There were THREE studies and THREE observation questions. Don’t forget to take your belongings with you when you leave the lab. Please clean up when you are finished.